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Correlational analysis of turbulent channel flows with injection

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Abstract—A turbulent quasi-stabilized flow in a flat channel with uniform injection is considered. Results are presented from calculations carried out on the basis of equations from the theory of the Kutateladze and Leontiyev limiting laws modified to include the effect of the longitudinal pressure gradient originating in such kind of flow. Based on these results a dependence of the relative friction factor on the injection parameter has been suggested. In addition, approximating formulae, that generalise the familiar solutions, are presented for calculating the longitudinal velocity profiles and longitudinal pressure distribution.

Consideration is given to a quasi-stabilized isothermal incompressible turbulent flow in a channel with homogeneous uniform (along its length) external medium injection through the channel walls. The inflow of extra mass to the channel causes the appearance of a longitudinal favourable pressure gradient, the larger the value of which, the higher the rate of injection. The originating pressure gradient exerts a marked effect on flow characteristics and it should be taken into account already for $V_w/U_o \approx 0.006$ [1, 2]. It is worth noting that, in contrast to the problem of external flow past a wall with injection, where a longitudinal pressure distribution is assumed, in the case considered it is the unknown function which depends on both the intensity of the injection through the walls and the flow being formed. Another special feature of the channel with injection is the presence of a transverse velocity component which grows with injection intensity. The effect of this velocity must be taken into account when selecting a turbulence transfer model [3]. The indicated features significantly complicate the problem, and hamper the extension of computational methods generally developed for the problem of external flow past a wall with injection to the internal problem, i.e. a channel flow with injection.

As with any physical process, the study of channel flow with injection raises the problem of its governing criteria. According to the approximate analysis given by the present author in ref. [4], for quasi-stabilized turbulent channel flow with uniform injection such governing criteria are: the relative velocity of injection $V_w/\bar{U}$ or V_w/U_o and the Reynolds number $-\bar{U}h/\nu$. With these criteria taken into account, all of the presently available methods of calculation can be divided into two groups. The first group comprises solutions obtained on the basis of Navier–Stokes equations assuming that $\mu = 0$. These solutions give an adequate description of experimental results in the case of a high relative velocity of injection when the viscosity effect turns out to be small in comparison with the effect of injection and of the accompanying pressure gradient [5]. The viscosity effect grows with a decrease in injection intensity and becomes the governing one when $V_w/\bar{U} \to 0$. Correspondingly, with low injection velocity the problem is solved under the boundary layer approximation while the injection and pressure gradient parameters are considered to be small [3].

Thus, by now, solutions have been found for high and low levels of injection. The problem of moderate injection has not yet been studied adequately (as well as the very definition of injection degrees). The present author is aware of only one such work [3] dealing with this problem. Unfortunately, the approach employed in ref. [3] and the relations obtained, e.g. $\psi = \psi(b)$, where $b = (2/C_{f_o})(V_w/U_o)$, are rather complicated for direct application due to the fact that U_o is the unknown function of x. In this work an attempt has been made to solve the problem by a simpler method based on the theory of limiting laws [6]. A simple closed system of relations is presented which makes it possible to predict the major features of the flow.

The theory of limiting laws has been developed for a boundary layer. In the case of non-gradient flow on a plate with injection it describes the distribution of velocity ω and friction function ψ depending on the parameter b within the range $b \leqslant b_{kr}$, i.e. up to the point of displacement. In a non-gradient flow and with $Re \to \infty$ the value of b_{kr} is equal to 4. This corresponds to $V_w/U_o \approx 0.01$. In the presence of the favourable pressure gradient the b_{kr} value increases [7]. In channel flows the attendant favourable pressure gradient turns out to be so significant that the displacement effect does not appear at all. This conclusion follows from the 'non-viscous' solution characteristic for strong injections and justifies the

NOMENCLATURE

b	injection parameter
C_f	friction factor
$2h$	channel height
P	static pressure
Re	Reynolds number, $2\bar{U}h/v$
u, v	longitudinal and transverse velocity components
$U_0, \bar{U}$	maximum longitudinal velocity and longitudinal velocity averaged over the channel transverse cross-section
$\bar{U}_0$	mean flow velocity at porous channel inlet
x, y	longitudinal and transverse coordinates.
τ	shearing stress under given and standard (denoted by subscript 0 and taken in an impermeable smooth channel) conditions, $\bar{\tau} = \tau/\tau_w, \bar{\tau}_o = \tau_o/\tau_{ow}$
ψ	ratio of friction factors under given and standard (denoted by subscript '0' and taken in an impermeable smooth channel) conditions, C_f/C_{f_o}
ω	local velocity under given and standard (in an impermeable smooth channel) conditions, $u/U_o, \omega_o = (u/U_o)_o$.

Greek symbols

η	non-dimensional transverse coordinate, y/h
λ	pressure gradient parameters $(h/\tau_w)(dP/dx)$; $\lambda_0 = \lambda + 1$
μ, v	dynamic and kinematic viscosities
ρ	density

Subscripts

k, r	parameters at the point of flow displacement away from the wall in critical injection
n	values associated with transition to 'non-viscous' flow regime
w	conditions at the wall.

application of the theory of limiting laws to channel flows with injection when $V_w/U_o > 0.01$.

The governing equations of the limiting theory have been written with allowance for the fact that the representative parameter of the channel flow is the Reynolds number based on the mean flow velocity. The flow was assumed to be two-dimensional (rectangular channel) and stabilized, and the transverse velocity to be negligibly small. Specific features of the flow attributable to the shape of the channel cross-section were considered by this author in ref. [4]. Under the assumptions made, equations for determining the friction function and velocity profile can be written in the form:

$$\sqrt{\psi} = \int_0^1 \sqrt{\bar{\tau}_o/\bar{\tau}}\, d\omega,$$

$$\sqrt{\psi}\,(1-\omega_o) = \int_\omega^1 \sqrt{\bar{\tau}_o/\bar{\tau}}\, d\omega. \tag{1}$$

Here, ω_o is the velocity profile in an impermeable channel at the considered Re value. By analogy with the known approximations [6], for shearing stress distributions it is assumed that:

$$\bar{\tau} = (1-\eta) + (\lambda_o\eta + b_1\beta\omega)(1-\eta),$$

$$\bar{\tau} = 1-\eta \quad \beta = (U_o/\bar{U})^2. \tag{2}$$

Approximation of equation (2) satisfies the following conditions:

$$\bar{\tau} = \bar{\tau}_o \quad \text{at} \quad b_1 = 0 \quad \lambda_o = 0,$$

$$\bar{\tau} \approx 1 + \lambda\eta + b_1\beta\omega \quad \text{for} \quad \eta \to 0.$$

As noted above, the value of the negative pressure gradient originating in channels with injection depends on the injection intensity at the wall. Accordingly, the parameters λ and $b_1\beta$ entering into the expression for $\bar{\tau}$ are not independent.

With the known relation between λ and $b_1\beta$, the system of equations (1) simplifies and becomes one-parametric. In the present work, the quantity $b_1\beta$ has been taken as an independent parameter, and the relation between λ and $b_1\beta$ has been obtained from the expression for static pressure distribution in a channel with injection, according to which:

$$-\frac{dP}{dx} = \frac{\tau_w}{h} + \beta\rho\bar{U}\frac{d\bar{U}}{dx}, \tag{3}$$

where in the case of uniform injection through the walls

$$\bar{U} = \bar{U}_o + V_w(x/h).$$

Relation (3) correlates the solutions obtained by quasi-one-dimensional [6] and two-dimensional [5] approaches to the problem at hand, which were valid for low- and high- ('non-viscous' flow) level injections, respectively. In fact, in the quasi-one-dimensional approximation, the equation of motion:

$$u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = \frac{\partial \tau}{\partial y} - \frac{dP}{dx},$$

integrated over the channel cross-section yields:

$$-\frac{dP}{dx} = \frac{\tau_w}{h} + k\rho\bar{U}\frac{d\bar{U}}{dx}, \tag{3'}$$

where:

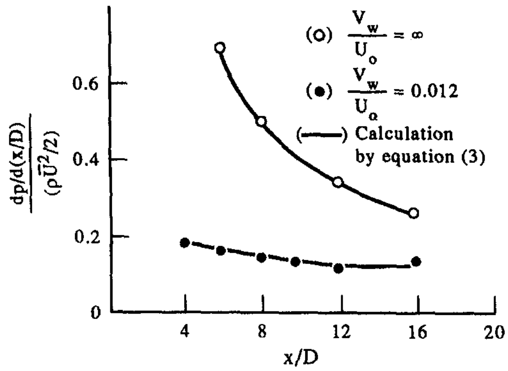

Fig. 1. Static pressure distribution along the channel length.

$$k = (U_o/\bar{U})^2 \int_o^1 \omega \, d\xi.$$

When solving two-dimensional Navier–Stokes equations for channel flow with strong injection according to ref. [5]:

$$\frac{dP}{dx} = \beta\rho\bar{U}\frac{d\bar{U}}{dx}. \quad (3'')$$

The pressure distribution in the form of equation (3) for strong injection, when the effect of friction on the flow becomes comparatively small (the term $\tau_w/h \to 0$), changes to expression (3″). Simultaneously, it satisfactorily describes the experimental data of ref. [4] and also of ref. [8] (Fig. 1) for small and moderate injections and practically coincides with equation (3′). The coincidence seems to be attributed to the fact that under these conditions the velocity profile is highly peaked and the value of the integral $\int_o^1 \omega \, d\xi$ differs little from 1.

From equation (3), after dividing all the terms by τ_w/h, we obtain:

$$-\lambda = 1 + b_1\beta^{3/2}. \quad (4)$$

As calculations have shown, the substitution of equation (4) into equation (2) leads to negative shearing stresses at values of η close to 1. Since τ cannot take values smaller than zero and $\beta^{3/2} \approx \beta$ (the difference between these quantities reaches the maximum value in a 'non-viscous' flow and does not exceed 23%), from equation (4) we obtain:

$$-\lambda \sim 1 + b_1\beta.$$

Finally, it gives:

$$\tilde{\tau} = (1-\eta)[1 + b_1\beta(\omega-\eta)]. \quad (5)$$

The distribution of $\tilde{\tau}$, equation (5), is quite well confirmed by the experimental data of ref. [8] (see Fig. 2).

With equation (5) taken into account, the system of equations (1) allows one to calculate the friction function ψ and longitudinal velocity distribution ω from the known $\omega(Re)$ and $\beta(V_w/U_o)$ (or $b_1\beta$). As seen from Figs. 3 and 4, in doing so one can find such pairs

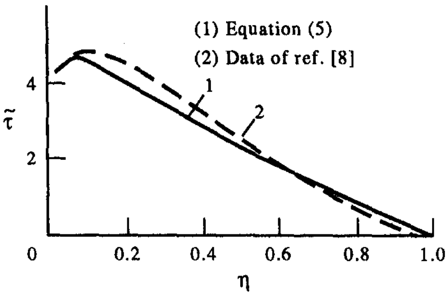

Fig. 2. Shearing stress distribution along the channel height at $V_w/U_o = 0.007$.

of the values of Re and $\beta(V_w/U_o)$, at which solution (1) is close to the solution for a 'non-viscous' flow:

$$u/U_o = \cos[\pi/2(1-\eta)]. \quad (6)$$

The viability of the system of equations (1) was checked by calculating the distributions of velocity ω for the conditions of experiments [1, 9]. The solution was sought by an iteration method. It was assumed that in the first approximation $\omega = \omega_o = \eta^{1/n}$, i.e. coincides with velocity distribution in an impermeable channel at a given value of Re; $b_1\beta = (2/C_{f_o})(V_w/U_o)$, C_{f_o}, was calculated by the Blasius law [10]. The values of U_o, V_w and β were taken from corresponding experiments. The calculation procedure amounted to the following. The value of ψ was calculated from the first equation of system (1). To find the distribution of velocity ω, the second equation of system (1) was written in the form:

$$\eta_i = \left(1 - \frac{\int_{\omega_i}^1 \sqrt{\tilde{\tau}_o/\tilde{\tau}} \, d\omega}{\sqrt{\psi}}\right)^n.$$

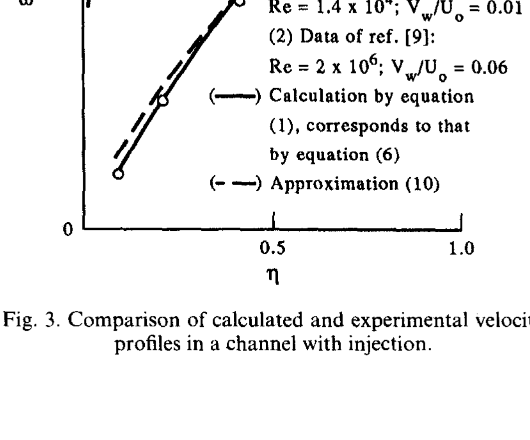

Fig. 3. Comparison of calculated and experimental velocity profiles in a channel with injection.

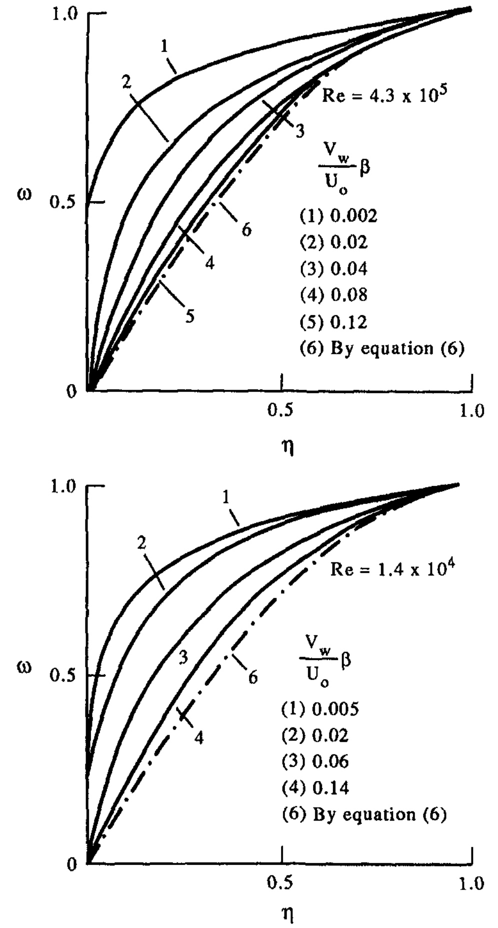

Fig. 4. Calculated velocity profiles in a channel with injection at different Reynolds numbers.

This expression allows one to find η_i for any assigned value of ω_i. The function $\omega_i(\eta_i)$ obtained in this way was used on the next step in the calculation of ψ, etc.

In Fig. 3 the calculated results for velocity distributions are compared with the experimental data of refs. [1, 9]. For the data of ref. [9], additionally the solution for non-viscious fluid, equation (6), was plotted, which, in the opinion of the authors of ref. [9], correlated the results obtained in their work. From Fig. 3 it follows that under the condition considered, including those for a 'non-viscous' flow, the calculated and experimental velocity distributions closely coincide.

Figure 4 presents calculated profiles corresponding to different values of $\beta(V_w/U_o)$ for two values of *Re*. Presented there is also relation (6). Using these data, the value of $\beta(V_w/U_o)$ can be determined which divides the regions of viscous and non-viscous flows. As seen from Fig. 4, the value of $\beta(V_w/U_o)_n$ at which the flow becomes non-viscous depends substantially on the value of *Re*. This value decreases with an increase in *Re*. A similar conclusion can be drawn on the basis of numerical investigations [3], according to which the value of the complex b_n is practically constant.

The ranges of the parameters within which calculations of the friction function ψ were conducted are: $Re_e = 1.4\times10^4$–2.0×10^6; $\beta(V_w/U_o) = 0.002$–0.006. The obtained values of ψ were presented in the form:

$$\psi = \psi(b_2) \quad b_2 = \frac{2}{C_{f_0}}\frac{V_w}{U}. \tag{7}$$

The selection of the quantity b_2 rather than $b_1\beta$ or $\beta(V_w/U_o)$ as a correlating parameter was made for the convenience of practical application. The coefficient β was attained from the corresponding velocity profile predicted from equations (1).

The calculations have shown (Fig. 5) that in the considered range of the *Re* and $\beta(V_w/U_o)$ values, the friction functions, presented in the form of the dependence on the parameter b_2, lie on a single curve which can be described by the formula:

$$\psi = \left(1 + \frac{b_2}{b_2 + 1}\right). \tag{8}$$

Figure 5 also presents a comparison of equation (8) with the experimental data of ref. [8] and with the results obtained in numerical calculations from the suggested by Yeroshenko and Zaichik [3] for $Re = 10^4$ and 10^5 and constructed under the assumption that $b_2 \approx b$. It follows from the figure that approximation (8) presents a qualitatively true description of experimental values of ψ. The conditions of the 'non-viscous' flow based on calculated data of ref. [3] are realized at the values of $b_2 \approx 7$–8 (Fig. 5). According to equation (8), in this case the value of ψ_n is equal to ~ 0.1. It should be noted that both the initial system of equations (1), and formula (8) obtained on their basis hold true only at sufficiently large Reynolds numbers.

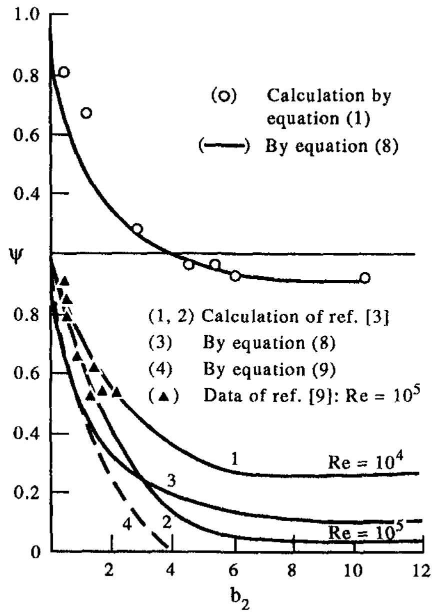

Fig. 5. Friction function vs injection parameter.

Of some interest is the comparison of approximation (8) with the relation:

$$\psi = (1-b_2/4)^2, \qquad (9)$$

which was obtained in ref. [6] on the basis of the limiting theory for the non-gradient flow conditions. From equation (5) it is clear that the divergence of the curves plotted from equations (8) and (9) becomes notable under the effect of the favourable pressure gradient at $b_2 \approx 1.5$; for $b_2 < 1.5$ the curves practically coincide.

Thus, based on the calculations carried out, it is shown that the Kutateladze–Leontiyev theory can be applied in principle for describing quasi-stabilized turbulent channel flows with uniform injection up to such rates when the 'non-viscous' solution can be used and a relation for predicting the friction function can be obtained.

The system of equations (1) can be used in practice for calculating velocity profiles only if the static pressure distribution along the channel is known. In a general case, it is unknown and, in its turn, according to equation (3), it depends on velocity distribution. To calculate the velocity profile in a channel with injection, it is suggested that one uses the following approximated relation:

$$\omega = \omega_0 - (\omega_0 - \omega_n)\left(\frac{b_2}{b_2+1}\right) \qquad (10)$$

where ω_n is the velocity distribution for a 'non-viscous' flow. This relation was constructed on the basis of the solutions corresponding to the flow in an impermeable channel and with strong injection, and also on the basis of the complex $b_2/(b_2+1)$ introduced above. Figure 3 illustrates the comparison between equation (10) and experimental data. Relations (3), (8) and (10) represent a closed system permitting one to calculate the basic hydrodynamic characteristics of a quasi-stabilized turbulent flow in channels with injection.

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